# MACs, Passwords and Asymmetric encryption

CS642: Computer Security



Asymmetric encryption



The RSA algorithm

Digital signing & public-key infrastructure

University of Wisconsin CS 642

## The Fundamental Equation

# $Z = Y^X \mod N$

#### **One-Way Functions**

# $Z = Y^X \mod N$

When X is unknown, the problem is known as the *discrete logarithm* and is generally believed to be hard to solve.

## **One-Way Trap-Door Functions**

# $Z = Y^X \mod N$

This equation is solvable for Y if the factorization of N is known, but is *believed* to be hard otherwise.

<u>Alice</u> <u>Anyone</u>

#### <u>Alice</u>



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  Y<sup>X</sup> mod N.
- Send Z and X to Alice.

#### <u>Alice</u>

- Select two large random primes *P* & *Q*.
- Publish the product N = PQ.
- Use knowledge of
  *P* & *Q* to compute *Y*.

#### Anyone

- To send message Y to
  Alice, compute Z =
  Y<sup>X</sup> mod N.
- Send Z and X to Alice.

When N = PQ is the product of distinct primes,  $Y^X \mod N = Y$ whenever  $X \mod (P-1)(Q-1) = 1 \text{ and } 0 \le Y < N.$ 

When N = PQ is the product of distinct primes,  $Y^X \mod N = Y$ whenever  $X \mod (P-1)(Q-1) = 1 \mod 0 \le Y < N$ . Alice can easily **select** integers *E* and *D* such that  $(E \times D) \mod (P-1)(Q-1) = 1$ .

Encryption:  $E(Y) = Y^E \mod N$ . Decryption:  $D(Y) = Y^D \mod N$ .

$$D(E(Y))$$
  
=  $(Y^E \mod N)^D \mod N$   
=  $Y^{ED} \mod N$   
=  $Y$ 

In practice, the encryption exponent *E* is almost always fixed to be  $E = 65537 = 2^{16} + 1.$ 

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 $D = (1 \div E) \mod (P - 1)(Q - 1).$ 

# **Public-Key Directory**

<u>Name</u>	Public Key	<b>Encryption</b>
Alice	N <sub>A</sub>	$E_A(Y) = Y^E \bmod N_A$
Bob	N <sub>B</sub>	$E_B(Y) = Y^E \bmod N_B$
Carol	N <sub>C</sub>	$E_C(Y) = Y^E \bmod N_C$
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	:	:

(Recall that *E* is commonly fixed to be E = 65537.)

An additional property

# An additional property $D(E(Y)) = Y^{ED} \mod N = Y$

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Only Alice (knowing the factorization of N) knows D. Hence only Alice can compute  $D(Y) = Y^D \mod N$ .

#### An additional property

$$D(E(Y)) = Y^{ED} \mod N = Y$$
$$E(D(Y)) = Y^{DE} \mod N = Y$$

Only Alice (knowing the factorization of N) knows D. Hence only Alice can compute  $D(Y) = Y^D \mod N$ .

This D(Y) serves as Alice's signature on Y.



#### \* Problem: How does a client get the public key for a website?



#### **Certificate Authority**







# cert signing





**рк**са, **sk**са

\* What does having a trusted TLS certificate prove?

- -That someone paid at least \$0
- –Proved to an intermediate CA that they controlled a given domain name for at least 5 minutes
- –If TLS established, proves they know the corresponding private key to the pub key in cert
- \* What could possibly go wrong?
  - -Any CA secret key in chain could be compromised
  - -Server secret key could be compromised
  - -Typo-squatting domain (gmal.com)
  - -Malicious root CA key installed on client
  - -DNS chicanery during verification process

# certificates



ClientHello, MaxVersion, Noncec, Supported ciphersuites

ServerHello, Version, Nonces, SessionID, Ciphersuite



blog.com

Certificate = (pks, domain name, signature, cert chain)

E(pks, PMS)

MS <- HMAC(PMS, "master secret" || Nc || Ns ) K1,K2 <- HMAC(MS, "key expansion" || Ns || Nc )

Change to symmetric cipher

ChangeCipherSpec, Finished, HMAC(MS, "Client finished" || H(transcript))

ChangeCipherSpec, Finished, HMAC(MS, "Server finished" || H(transcript'))

Exchange info using Ek1, Ek2

# DigiNotar

- Dutch CA DigiNotar compromised in 2011
- Attackers generated fake certificates
- Twitter.com was redirected to fake site
- Attackers eavesdropped with man-in-the-middle attacks
  - Iranian govt eavesdropping on dissidents





# DigiNotar

How did compromise occur?

DigiNotar had crappy security

- Out-of-date antivirus software
- Poor software patching
- Weak passwords
- No auditing of logs
- Poorly designed local network

## eDellRoot

- Dell shipped several computer systems with a selfsigned root CA certificate preinstalled
  - The cert also contained the CA secret key
- Intended purpose: something to do with automated support software
- If certificate removed, automatically reinstalls on reboot



#### eDellRoot